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THE CONDUCTION OF HEAT FROM SLIDING SOLIDS

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Abstract—The large scale restrictions to heat flow from two sliding solids can have a significant effect on the temperature field near the interface. It is shown that a practical system can be approximated to two semi-infinite solids whose temperatures at infinity are related to the heat flow rates through them. A number of existing semi-infinite solid solutions are generalised to allow for a difference between the temperatures at infinity and a new particular solution is developed for the case of sub-surface heat generation. The method is then extended to situations with several contact areas and the effect of geometrical and physical properties on the interfacial boundary conditions is discussed.

NOMENCLATURE

a, radius of an actual contact area;
b, radius of the nominal contact area;
c, specific heat;
Q_1, Q_2, heat flow into solids 1, 2;
Q_T, total rate of heat generation;
q_i, heat input to the ith contact area;
n, total number of actual contact areas;
r, distance from the heat source;
T_1, T_2, temperatures of the boundaries (or at infinity) of the solids 1, 2;
t, time;
V, relative velocity;
x, projection of the distance r onto the direction of relative motion;
z, perpendicular distance from the surface;
z_0, depth of a sub-surface heat source;
κ, thermal diffusivity;
ψ, fractional reduction in constriction resistance due to the finite size of the nominal contact area;
ρ, density;
θ_1, θ_2, temperature produced at the contact area by a unit heat input to solids 1, 2;
θ_{s1}, temperature produced at the contact area by a unit sub-surface heat input to solid 1.

1. INTRODUCTION

It is now generally accepted that one of the most important parameters in all sliding contact systems is the temperature field in the vicinity of the sliding surfaces. This can influence the process in a variety of ways, notably by controlling the oxidation rate at the interface, the adsorption of gases and lubricants, the mechanical properties of the surface layers and the viscosity of the lubricant.

A number of solutions to the relevant heat conduction problem have been published, but these have been largely concerned with the ideal case of two semi-infinite solids, with zero temperatures at infinity, making contact at a single area in the interfacial plane. These solutions have frequently been used out of context and the present paper demonstrates some of the changes produced in the temperature field by the practical limitations of finite systems and large scale thermal restrictions.

The boundary conditions in a practical sliding system do not generally admit an exact analytical solution and a choice has to be made between a numerical solution (e.g. by a relaxation or finite difference equation method) and the use of a physical and/or analytical approximation. The latter method is used here, not because it is necessarily the best method to use in all circumstances, but because it enables the effect of the boundary conditions on the solution to be presented with greater generality. However, it is worth remarking that the greatest limitation on accuracy is that imposed by the absence of reliable information about the boundary conditions at the interface. There is therefore little justification for obtaining a high numerical accuracy for a particular solution. However, accurate solutions are necessary for assessing the reliability of the approximate methods.

It is shown how an approximate solution to a practical problem can be derived from a knowledge of the restrictions to heat flow from the remote surfaces of the solids and the effect of changes in the temperatures at infinity in the semi-infinite solids solution appropriate to the microscopic boundary conditions at the interface. In Section 4, a number of particular solutions to the latter problem are derived from existing 'equal-temperatures-at-infinity' solutions and a new solution is obtained for the particular case where heat is generated below the surface of one of the solids. The discussion is then extended to situations with several contact areas whose temperature fields interact (Section 5).

The division of frictional heat between the solids proves to be very sensitive to the motion of the contact area relative to the surfaces. In practice, this relative motion is determined by the nature of the individual interactions between the asperities of the surfaces. The solutions of the heat conduction problem are therefore interpreted in their physical context in the Discussion.

2. BOUNDARY CONDITIONS OF THE PROBLEM

The essential features of sliding systems which influence the heat flow are the nature of the contact between the solids, the distribution of heat sources and the large scale cooling effects at the non-contacting surfaces.

2.1 *The area of contact*

The area of each surface within which contact is possible will be defined by the geometry of the sliding system; this is known as the *nominal contact area.* However, the solids will not necessarily be in actual contact continuously over the entire nominal area. Bowden and Tabor and others have shown that the roughness of surfaces restricts their *actual contact* to a number of small areas at the peaks of the surface asperities. As the load is increased, these asperities deform, allowing the solids to move closer together and causing more contact areas to be formed. A recent account of the theory of the contact of stationary solids has been published by Greenwood [1]. There is a considerable amount of evidence to suggest that the contact of sliding solids is also restricted to a number of small areas. However, in this case the distortion produced by the relative motion of the solids will cause the actual contact areas either to be transient due to the fracture of the junctions, or to move relative to one or both solids.

2.2 *The location of the heat sources*

The heat generated during sliding is associated with the plastic deformation caused by mechanical interaction between the solids at or near the actual contact areas. The distribution of the heat source therefore depends on the nature of these interactions. Some indication of the location of plastic deformation is given by the studies of large scale model junctions by Green [2], Greenwood and Tabor [3] and Brockley and Fleming [4]. Their work supports the general proposition that most of the heat is generated within the sphere of which the actual contact area forms a diametral plane.

2.3 *Large scale cooling effects*

It is sometimes argued that, if the distance between the cooled boundaries of the solids and the interface is large in comparison with the dimensions of the actual contact regions (as is generally the case), it is possible to approximate the system to two semi-infinite solids. It is true that the temperatures at such remote regions of the semi-infinite solids are scarcely affected by the conditions at the interface. However, the material beyond these imaginary boundaries performs the function of an infinite heat sink of negligible thermal resistance. In a finite system in a steady thermal state, this infinite heat sink is replaced by a practical heat-transfer process which causes the average temperature of the boundaries to be related to the heat flow from them.

Thus

$$Q_1 = \mathscr{F}_1(T_1) \quad (1)$$

$$Q_2 = \mathscr{F}_2(T_2) \quad (2)$$

where Q_1, Q_2 are the rates of heat flow through the solids 1, 2 respectively and T_1, T_2 are the temperatures of their exposed surfaces.

In the contact of two semi-infinite solids, the temperature at infinity is unaffected by any changes in the temperature at the contact area (assumed finite), but the temperature at infinity does affect the temperature field near the contact area. Similarly, even if the boundaries of two finite solids are sufficiently remote from the interface to be unaffected by the contact conditions there, the temperature field near the contact area and the distribution of heat between the solids will be affected by the temperature of the boundaries. In fact, this practical system can be approximated to two semi-infinite solids with the same contact conditions, but of which the temperatures at infinity are related to the heat flow rates by equations (1) and (2).

3. METHOD OF SOLUTION

In general, the relationships given by equations (1) and (2) will be known or may be found by independent experiments on the solids. Also, the total rate of heat generation at the interface (Q_T) will be determined by the mechanical conditions at the interface and

$$Q_1 + Q_2 = Q_T. \quad (3)$$

In order to produce a solution, we also need to find the effect of the unknown 'temperatures at infinity' (T_1, T_2) and the known interfacial contact and heat generation conditions on the distribution of Q_T between two semi-infinite solids. This will provide a fourth equation relating the four unknowns (Q_1, Q_2, T_1, T_2). This relationship can be obtained by superposing two subsidiary solutions:

(1) with the same boundary conditions except that the temperatures at infinity are both zero,

(2) with non-zero temperatures at infinity (T_1, T_2), but with no heat generation at the interface.

This approach allows us to make use of existing solutions to problem (1). The effect of various contact conditions on these semi-infinite solid solutions will be discussed in section 4.

4. THE SEMI-INFINITE SOLID SOLUTION

The temperature (T) in an infinite solid, initially at zero temperature, due to an instantaneous point source of heat at time $t = 0$ is subsequently given by

$$T = \frac{Q}{8\rho c\sqrt{(\pi\kappa t)}} \exp\left(-\frac{r^2}{4\kappa t}\right) \quad (4)$$

where Q is the quantity of heat liberated, r is the distance from the source and the solid has density ρ, specific heat c and thermal diffusivity κ (Carslaw and Jaeger [5]).

Since the infinite solid is spherically symmetrical about the point source, there can be no heat flow across a diametral plane and the temperature reached in a semi-infinite solid with an instantaneous point source on the surface and no heat loss from the surface is

$$T = \frac{Q}{4\rho c\sqrt{(\pi\kappa t)}} \exp\left(-\frac{r^2}{4\kappa t}\right) \quad (5)$$

since all the heat is now directed into one half space.

This solution may be extended by integration to any distribution of heat input on the surface of a semi-infinite solid and it is the basis of most of the work on heat conduction during sliding.

Blok [6] and Jaeger [7] have both produced approximate solutions to the problem of two solids in relative motion with a continuous heat source uniformly distributed over the area of contact, the latter being stationary with respect to one solid. Circular, square and band heat sources were considered all of which gave very similar results. The solids were both assumed to be semi-infinite with zero temperature at infinity.

The problem was solved by finding the temperature at the contact area due to a uniformly distributed heat input to each solid, by integration of equation (5). The distribution of heat between the surfaces was then adjusted to equalise these temperatures. Thus, if a unit heat input causes the temperature of the contact area in solids 1 and 2 to be θ_1 and θ_2 respectively, the actual heat flow into the solids will be

$$Q_1 = \frac{\theta_2 Q_T}{(\theta_1 + \theta_2)} \tag{6}$$

$$Q_2 = \frac{\theta_1 Q_T}{(\theta_1 + \theta_2)}. \tag{7}$$

These equations satisfy equation (3) and cause the temperature at the contact area in each solid to be $\theta_1\theta_2 Q_T/(\theta_1 + \theta_2)$, the equality being required for continuity of temperature through the contact.

It is not strictly correct to specify a uniformly distributed heat input to each solid since, although the heat source is assumed to be uniform, the distribution of heat between the solids will vary over the contact area. As a consequence of this approximation, it is not possible to match the temperature over the entire contact area: Blok equates the maximum temperatures at the interface whilst Jaeger equates the average temperature over the contact area. The latter method takes some account of the variation of temperature over the contact area and is thus less likely to fail under unusual conditions.

A more exact solution to this problem has recently been produced by Cameron *et al.* [8] by matching the temperature at all points in the contact area by a numerical method and thus allowing for the variation of heat distribution with position. They also solve for the case where the contact area moves relative to both solids. They conclude that the solutions of Blok and Jaeger are remarkably accurate in spite of the approximate method (Symm [9]). In view of the additional complications introduced by the more exact solution, it is reasonable to use the approximate method unless a high accuracy is required.

The solution to the parallel problem with non-zero temperatures at infinity and no heat generation can be found from Jaeger's results by a similar temperature matching process in which the heat flow in the hotter solid is reversed. Thus it follows from the above definitions that a temperature difference of $(\theta_1 + \theta_2)$ between the remote boundaries of the two solids will cause a unit heat flow between them, within the limits imposed by the Jaeger approximation.

Combining these results we can derive the following equations for Q_1, Q_2 when the temperatures at infinity are T_1, T_2

$$Q_1 = \frac{\theta_2 Q_T + (T_2 - T_1)}{(\theta_1 + \theta_2)}, \tag{8}$$

$$Q_2 = \frac{\theta_1 Q_T + (T_1 - T_2)}{(\theta_1 + \theta_2)}. \tag{9}$$

It should be noted that the quantities θ_1, θ_2 are temperature differences *per unit heat flow* so dimensional consistency is preserved. They depend on the shape of the contact area and its velocity relative to the solid. Mathematical expressions and numerical values can be found in Blok [6] and Jaeger [7].

The temperature at the actual contact area can be found from equations (8) and (9) and is

$$T = \frac{(T_1\theta_2 + T_2\theta_1)}{(\theta_1 + \theta_2)} + \frac{\theta_1\theta_2 Q_T}{(\theta_1 + \theta_2)}. \tag{10}$$

4.1 *Sub-surface heat generation*

So far it has been assumed that the heat is generated at the interface between the solids. In practice, the plastic deformation which results in heat generation extends to some depth below the surface. The effect of sub-surface heat generation has been described qualitatively by Ling [10], Ling and Pu [11] and Barber [12], the former using it as an explanation of a temperature difference between the solids which was observed experimentally by Ling and Simkins [13]. In fact, a temperature difference can occur in the absence of sub-surface heat generation as will be shown in Section 6.1, but the exact location of heat sources becomes particularly important if the metallic contact at the interface is reduced by oxide layers.

Consider a sliding system consisting of two semi-infinite solids (1 and 2) in contact at a single circular area, with a continuous unit heat source below the surface of solid 1 at a point which is stationary relative to the contact area. Let all the heat generated flow into solid 1 (i.e. none flows through the contact area into solid 2) and let the temperature at infinity in solid 1 (T_1) be zero. In the Appendix, an expression is derived which gives the average temperature (θ_{s1}) produced at the contact area in these circumstances as a function of the position of the sub-surface heat source and the velocity and radius of the contact area. Since we have specified that there is no heat flow into solid 2, its temperature must be uniform (except in the region close to the contact area) and equal to θ_{s1}. Thus, the temperature at infinity in solid 2 (T_2) is equal to θ_{s1}.

The heat flow solution for general values of T_1 and T_2 is obtained by superposition of a heat flow (Q_2) through the contact area. Thus

$$Q_2 = \frac{T_1 - (T_2 - \theta_{s1}Q_T)}{(\theta_1 + \theta_2)}$$

$$= \frac{\theta_{s1}Q_T}{(\theta_1 + \theta_2)} + \frac{(T_1 - T_2)}{(\theta_1 + \theta_2)} \quad (11)$$

and the heat flow into solid 1 (Q_1) is

$$Q_1 = Q_T - Q_2$$
$$= \frac{(\theta_1 + \theta_2 - \theta_{s1})Q_T}{(\theta_1 + \theta_2)} + \frac{(T_2 - T_1)}{(\theta_1 + \theta_2)}. \quad (12)$$

The temperature at the contact area is

$$T = T_2 + \theta_2 Q_2$$
$$= \frac{(T_1\theta_2 + T_2\theta_1)}{(\theta_1 + \theta_2)} + \frac{\theta_{s1}\theta_2 Q_T}{(\theta_1 + \theta_2)}. \quad (13)$$

Equations (8)–(10) can thus be considered as special cases of equations (11)–(13) in which $\theta_{s1} = \theta_1$. i.e. for which the heat source is uniformly distributed over the contact area. The results derived in the Appendix are therefore plotted in Fig. 1 in the form θ_s/θ to demonstrate the effect of the location of the heat source on the distribution of heat between the solids. In the particular case where $T_1 = T_2$, equation (11) shows that Q_2 is directly proportional to θ_{s1}. Thus, the effect of removing the heat source from the contact area to a position below the surface of solid 1 is to reduce the heat flow into solid 2 in the proportion θ_{s1}/θ_1.

Figure 1 shows that the location of the heat source becomes critical at high speeds, but if $Va/4\kappa$ is less than 1 and most of the heat is generated near to the contact area, the surface heat source solution will probably be reasonably accurate.

The heat flow due to a general distribution of heat sources in one or both solids can be found by a suitable integration of θ_s. The temperatures (θ_{s1}, θ_{s2}) at the contact area in each solid are found on the assumption that no heat flows through it. Since the temperature at the contact area must be continuous, the temperatures at infinity must differ by ($\theta_{s2} - \theta_{s1}$). The solution is then generalised by superposing a general heat flow through the contact area.

It should be noted that the velocity V which occurs in equation (A4) is the velocity of the source *relative to the solid in which it is situated.* Thus V will not generally be the same for the two solids, but the algebraic difference of the velocities will equal the relative velocity of the solids.

4.2 *Transient contact areas*

Under certain circumstances, the actual contact areas between the solids (and consequently the heat sources) will be transient, in which case the foregoing analysis is not exact. However, it will still be acceptable as an approximation provided that a steady thermal state is reached at an early stage of the contact cycle. This condition is satisfied if $a/\sqrt{(4\kappa t)} \ll 1$, where a is the radius of the contact area, t is its duration and κ is the thermal diffusivity of the material.

Short duration contacts can be produced by geometrical limitations of the sliding system. Thus, if one of the two sliding solids is small, an asperity in the other surface will only be able to plough through it for a short distance and the actual contact areas will be of short duration. However, a more important transient contact mechanism is that associated with a shearing interaction. If the two solids have similar mechanical properties, the typical interaction will not be the ploughing of one solid by the other, but the symmetrical distortion of adhesive junctions between the solids culminating in their fracture. A steady state is maintained since new junctions are formed continuously. The duration of such a junction is limited by the maximum strain which it can endure before fracture. From a thermal point of view, the most significant difference between this mechanism and that of ploughing is that the adhesive junction distorts symmetrically with respect to each solid. Furthermore, if we regard the plastic distortion and consequently the heat source as being localised in a small volume of metal connecting the surfaces, the heat input to each surface is

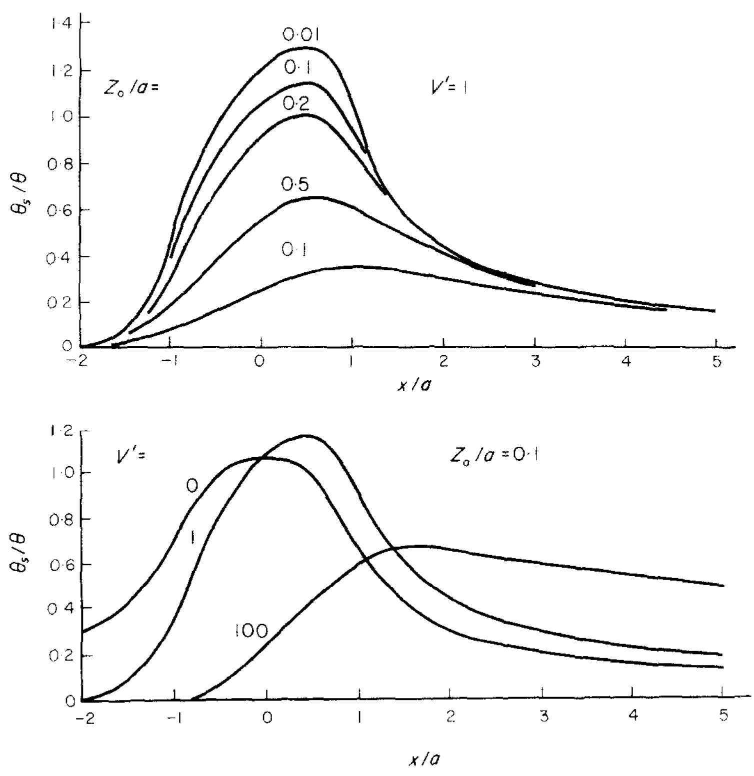

FIG. 1.

localised in the area of attachment of the junction and this is approximately stationary in the surface. The relative motion is taken up by the distortion of the junction.

The transient heat flow problem presents analytical difficulties, but an approximate solution has been obtained for the range in which t is small [i.e. when $\exp(-a^2/4\kappa t) \ll 1$] and is described more fully in Barber [14]. The basic results are as follows.

If the temperatures of the solids at infinity are equal ($T_1 = T_2$), the heat generated at a transient contact is distributed in such a way that

$$\frac{Q_1}{Q_2} = \frac{\rho_2 c_2 \sqrt{\kappa_2}}{\rho_1 c_1 \sqrt{\kappa_1}}. \qquad (14)$$

The same result is obtained for high speed, steady state sliding, if the contact area moves symmetrically with respect to both solids. However, at low speeds the steady state solution is

$$\frac{Q_1}{Q_2} = \frac{\rho_2 c_2 \kappa_2}{\rho_1 c_1 \kappa_1}. \qquad (15)$$

If $T_1 \neq T_2$, there will also be a superposed heat flow through the contact area, the average value of which is approximately

$$Q = \frac{8a^2(T_1 - T_2)}{\sqrt{(\pi t)}}\left(\frac{\rho_1\rho_2 c_1 c_2 \sqrt{(\kappa_1\kappa_2)}}{\rho_1 c_1\sqrt{(\kappa_1)} + \rho_2 c_2\sqrt{(\kappa_2)}}\right) \qquad (16)$$

where t is the duration of the contact. The corresponding low speed steady state solution is

$$Q = 4a(T_1 - T_2)\left(\frac{\rho_1\rho_2 c_1 c_2 \kappa_1\kappa_2}{\rho_1 c_1\kappa_1 + \rho_2 c_2\kappa_2}\right). \qquad (17)$$

However, at high speeds, an equation similar to equation (16) is obtained except that the duration of the contact (t) is replaced by a multiple of a/V (i.e. by the time taken for the contact area to traverse a distance comparable with its radius). This similarity between the short duration transient and steady state, high speed solutions results from the fact that, in each case, the heat flow in all directions except that normal to the surface can be neglected during the period when the contact area is overhead. The results demonstrate the overriding importance of the relative motion of the contact area and the solids in high-speed sliding heat flow problems.

5. SYSTEMS WITH SEVERAL ACTUAL CONTACT AREAS

In general, two sliding solids will make contact at several actual contact areas, the number and size distribution of which will depend on the profiles of the surfaces and the applied load. If the contact conditions and the heat source distributions are specified, the problem is essentially similar to that of a single contact area with a non-uniform distribution of heat sources. A numerical solution could be obtained from a series of linear simultaneous equations by a suitable subdivision of the discontinuous contact area. This method was used by Chao and Trigger [15] to find the temperature distribution over the tool chip interface during metal cutting. However, when the actual contact areas are well separated, an approximate analytical solution can be obtained.

As a first approximation, suppose that there is no interaction between the temperature fields of adjacent contact areas. In this case, the solution is obtained by a suitable summation of equations (8) and (9) over the contact areas. Thus

$$Q_1 = \sum_{r=1}^{r=n} \frac{\theta_{2r} q_r}{(\theta_{1r} + \theta_{2r})} + (T_1 - T_2)\sum_{r=1}^{r=n}\frac{1}{(\theta_{1r} + \theta_{2r})} \qquad (18)$$

where θ_{1r}, θ_{2r} are the values of θ_1, θ_2 for the rth contact area and q_r is the heat generated at it per unit time. T_1 and T_2, the temperatures at infinity, are the same for all contact areas.

This approximation will fail if any two contact areas are separated by a distance which is not large in comparison with their linear dimensions, since, in this case, a heat input at one area will produce a significant rise in temperature at the other. In order to take account of this effect, we need to replace equation (16) by n equations relating the heat flow rates (q_r) to the temperatures at the individual contact areas (T_r). Thus, the temperature at the rth contact area is the sum of the temperatures produced there by the heat flow from it into the solid, given by equation (8), and that due to the heat flow from all the other contacts. However, the second term is approximately equal to the temperature which would be produced at the same point by the same total heat input distributed continuously over the nominal contact area. Thus, the interaction between the temperature fields of adjacent heat sources may be regarded as the additional thermal resistance caused by the finite size of the nominal contact area. Greenwood [16] has examined the validity of this approximation for distributions of stationary contact areas of varying size and found that it is accurate to 2 per cent except for obviously singular distributions. The effect of actual contact area distribution is also discussed by Cooper *et al.* [17]. The method is best explained with the aid of an example.

Consider a number (n) of stationary contact areas each of radius a and with a heat input rate q, uniformly distributed over a circular nominal contact area of radius b on the surface of a semi-infinite solid. The temperature at a particular contact area can be found by superposing the n solutions for each heat input taken independently. However, only the temperature near a contact area is significantly affected by its radius. Thus, in finding the temperature at the rth contact area, we can approximate the remaining $(n-1)$ heat sources to equal sources uniformly distributed over circles of radius $b/\sqrt{n}$, concentric with the actual contacts. Now a set of n heat sources of radius $b/\sqrt{n}$, uniformly distributed over a circle of radius b is approximately equivalent to a single source of strength nq, uniformly distributed over the same circle. Thus, the temperature at the rth contact area can be obtained by superposing

(1) A heat input of strength nq uniformly distributed over the nominal contact area (radius b).

(2) A heat input of strength q at the rth contact area (radius a).

(3) A heat output of strength q uniformly distributed over a circle of radius $b/\sqrt{n}$ concentric with the rth contact area.

The sum of terms 1 and 3 is an approximation to the temperature produced at the rth contact area by the other $(n-1)$ heat sources.

The comparable problem of n equal stationary contact areas uniformly distributed over the end of a cylindrical conductor has been the subject of extensive study in the field of thermal contact conductance. The results are generally expressed in terms of the fractional reduction (ψ) in constriction resistance through a single contact caused by the finite size of the conductor. This result is then extended to the multiple contact problem by dividing the conductor into a bundle of parallel rods in such a way that the end of each rod contains one actual contact area and there is no heat flow between adjacent rods. The problem is thus resolved into one of n resistances in parallel, each of which is known as a particular example of the single contact solution. However, the function (ψ) must now be related to the radius of the individual rods; if the conductor has a radius b, this will be $b/\sqrt{n}$.

By analogy with the cylindrical conductor problem, we can define ψ for the semi-infinite solid as the fractional reduction in constriction resistance due to the finite size of the nominal contact area, i.e. the ratio of the temperatures of the rth contact area due to (3) and (2) above. Thus, since the constriction resistance is inversely proportional to the radius of the contact area,

$$\psi = \frac{a\sqrt{n}}{b}. \tag{19}$$

This result also provides a reasonable approximation for the cylindrical conductor of diameter b (cf. the results of Hunter and Williams [18]).

5.1 *Method of solution for solids with several contact areas*

When the solids make contact at several discrete areas, it is convenient to modify the method of solution outlined earlier by including the thermal resistance of the nominal contact area, term (1) above, in the equations (1) and (2) (i.e. as part of the large scale thermal resistance). The constriction alleviation term, term (3) above, is included in the semi-infinite solution. With these modifications, the only additional complication introduced by the multi-contact system is the replacement of a relationship such as equations (8) and (9) by a summation or integration over a known series of contacts such as equation (18). In fact, if all the contact areas and heat sources are similar, this equation merely becomes

$$Q_1 = \frac{n\theta_2 q + n(T_1 - T_2)}{(\theta_1 + \theta_2)}. \tag{20}$$

This procedure contains an implicit assumption: that the bulk temperature i.e. the temperature produced by the equivalent continuously distributed source—term (1) above $+T_1$, is constant over the nominal contact area. In practice, this assumption is not justified and we should take account of the variation of bulk temperature. However, the additional complexity introduced by this variation is not usually justified by the accuracy of our knowledge of the boundary conditions of the problem and it is therefore more reasonable to approximate the bulk temperature to its average value over nominal contact area. This is a 'large-scale' version of Jaeger's approximation referred to in the discussion of semi-infinite solid solutions (Section 4).

6. DISCUSSION

6.1 *The bulk temperature difference or temperature jump*

The irregularity in the temperature field produced by a heat input at an actual contact area is only significant in its immediate vicinity and a typical contact area is believed to be about 10^{-5} m dia. (see for example Rabinowicz [19]). Thus, the limitations on size and positioning of embedded thermocouples ensure that they will not respond to the temperatures at the actual contact areas, but will record the bulk temperature.

The temperature at an actual contact area must be the same in both solids (to maintain continuity), but it is not necessary for the bulk temperature of the two solids to be equal, or, for that matter, for T_1 to equal T_2. Thus, in general, if a thermocouple is embedded beneath the surface of each solid, there will be a difference between their temperature readings.

This temperature difference was observed by Ling and Simkins [13]. In more recent papers, Ling proposed an explanation based on subsurface heat generation. He said that '*Whenever the capacity of one of the bodies to remove heat away from the interfacial zone is less than the amount of heat generated on that surface, there will be a temperature jump across the interface*' (Ling and Pu [11], Ling [10]).

However, the existence of a temperature jump is not conclusive evidence in favour of subsurface heat generation. The bulk temperatures will be equal, i.e. there will be no temperature jump, only when the temperature field near to an actual contact area is similar to that in the corresponding semi-infinite solids with *equal temperatures at infinity*. Thus, a temperature jump is produced *whenever the ratio of the large scale thermal conductances away from the interface differs from the ratio of heat inputs which would be produced in the same solids if these large scale conductances were infinite.*

Suppose that one of the solids is completely insulated from the surroundings so that no heat can flow from it except through the interface. Let there be one area of actual contact and let all the heat be generated in the other, non-insulated solid. In the steady state, no heat will flow through the actual contact area and Ling's criterion predicts that there cannot be a temperature jump between the solids. However, since there is no heat flow at all in the insulated solid, its temperature will be uniform and equal to that at the contact area. There will therefore be a temperature jump between the solids unless the bulk of the non-insulated solid is also at the contact area temperature. This condition can only be satisfied if the heat source is not only below the surface, but sufficiently distant from the actual contact area for the temperature at the latter to be equal to the average temperature over the nominal contact area. This problem is also discussed by Johnson [20].

6.2 *The effect of material properties on the boundary conditions*

In order to apply the foregoing solutions to a practical sliding contact problem, we need to make some assumptions or observations about the microscopic boundary conditions at the interface. Thus, we need to know the number, size and duration of the actual contact areas, their velocities relative to the two surfaces and the location and strength of the associated heat sources. These factors are all determined by the nature of the mechanical interactions between the asperities of the surfaces.

It must be emphasised that, whereas the motion of the *nominal* contact area relative to the solids is determined by the geometry of the system, the motion of the *actual* contact areas is determined only by the nature of the deformation process*. Thus, if solid 1 is much harder than solid 2, its asperities will plough through solid 2 causing the actual contact areas to be stationary relative to solid 1 and to move relative to solid 2. The quantity θ decreases as the speed of the contact area over the surface increases so that in this case $\theta_1 > \theta_2$. Thus from equations (6) and (7), $Q_2 > Q_1$.

Furthermore, most of the plastic deformation (and hence the heat generation) will take place in the ploughed solid (2) and this fact also tends to increase the ratio Q_2/Q_1.

* The *duration* of a particular actual contact area is however influenced by the large scale geometry (see 4.2).

Thus, in an otherwise symmetrical sliding system, the greater part of the heat generated by friction should flow into the softer solid. If the system is finite and there is an equal resistance to heat flow from each solid, this will cause the softer solid to have the higher bulk temperature. The effect will become more noticeable at high sliding speeds, since θ decreases continuously with increasing speed.

Most metals become softer with increasing temperature so that this asymmetry in heat flow should exaggerate the difference in hardness between the two solids. This could conceivably lead to an instability in the sliding of similar solids. Thus, if either solid is initially hotter than the other, the consequent difference in hardness between them will cause an asymmetry in heat flow which tends to perpetuate the condition. No experimental evidence of such a process has been reported and it is only likely to occur if the rate of heat generation is sufficient to raise the bulk temperature of the solids into the range where thermal softening is significant. However, Manton *et al.* [21] have observed a comparable asymmetry in the heat flow from a lubricated rolling/sliding contact which they attribute to the effect of temperature on viscosity. This causes the location of maximum shear strain rate to be displaced from the centre of the lubricant film towards the hotter surface and hence increases the proportion of heat flow into the latter.

The mechanism of asperity interaction in the sliding of similar metals is difficult to predict. Initially, the asperities will probably adhere and distort symmetrically, but during sliding they will become work hardened and some ploughing will probably occur in both solids. However, the symmetry of the system requires that the heat generated should be divided equally between the solids if their bulk temperatures are equal.

7. CONCLUSIONS

Previous solutions to practical sliding contact heat flow problems have generally been based on the assumption that the system can be approximated to two semi-infinite solids with zero temperatures at infinity. The introduction of large-scale thermal resistances into the system generally produces a temperature difference between the boundaries of the two solids and a more reasonable approximation is that of two semi-infinite solids of which the temperatures at infinity are related to the heat flow through them. This modification has a relatively small effect on the temperature at the actual contact areas unless the restrictions to heat flow are large, but the division of heat between the solids is disturbed.

An approximate solution to a practical problem can be obtained from the following simultaneous equations

1. Two equations relating the temperature of the distant boundaries of each solid (T_1, T_2) to the heat losses from them (Q_1, Q_2).

2. An equation relating the temperatures T_1, T_2 (now considered as the temperatures at infinity in two semi-infinite solids) to the distribution between the solids of a known total heat source (Q_T).

The form of this 'semi-infinite' equation (2) depends on the boundary conditions at the interface. A number of existing solutions have been generalised to allow for a difference between the temperatures at infinity in the two solids and a new particular solution, for the case of sub-surface heat generation, is derived in the Appendix.

When there are several areas of actual contact, it has been shown that the interaction between their temperature fields is equivalent to the additional constriction resistance of the nominal contact area. In this case, the average temperatures over the nominal contact area (referred to here as the bulk temperatures) have to be equated to the temperatures at infinity in the semi-infinite solid solution and a correction has to be made to the latter to take account of the fact that the heat flow through the actual contact areas is only spreading out into a finite conductor. This correction is discussed by Hunter and Williams [18] and is described by them as the 'constriction alleviation factor'.

The division of heat between the solids is very sensitive to the motion of the actual contact areas relative to the solids. This motion depends upon the physical properties of the surfaces and it is deduced that, at high speeds, most of the heat will flow into the softer solid.

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APPENDIX

The temperature produced in an infinite solid by a point source moving through it at a speed V is obtained by integrating equation (5) and is

$$T = \frac{q}{4\pi\kappa\rho c r}\exp[-V(r-x)/2\kappa] \qquad (A.1)$$

where q is the rate of heat input and x is the projection of the length r onto the direction of motion of the heat source.

To extend this result to the semi-infinite solid, we introduce an identical source moving parallel to the first at a distance of $2z_0$, measured perpendicular to the direction of motion. The infinite solid is now symmetrical about the plane which perpendicularly bisects the line joining the two sources. The temperature field must also be symmetrical, thus no heat can flow across this plane and we have the solution for the temperature in a semi-infinite solid due to a point source moving parallel to the surface at a depth z_0.

$$T = \frac{q}{4\pi\kappa\rho c}\left[\frac{\exp\{-V[\sqrt{(x^2-y^2+z^2)}-x]/2\kappa\}}{\sqrt{(x^2+y^2+z^2)}} + \frac{\exp(-V\{\sqrt{[x^2+y^2+(z+2z_0)^2]}-x\}/2\kappa)}{\sqrt{[x^2+y^2+(z+2z_0)^2]}}\right] \qquad (A.2)$$

where x, y, z are the co-ordinates of the point relative to the source at any instant, x being measured in the direction of motion of the source, y in the direction perpendicular to x and parallel to the surface plane and z perpendicular to the surface plane.

In particular, the temperature at a point on the surface $(x, y, -z_0)$ is

$$T = \frac{q}{2\pi\kappa\rho c}\,\frac{\exp\{-V[\sqrt{(x^2+y^2+z_0^2)}-x]/2\kappa\}}{\sqrt{(x^2+y^2+z_0^2)}}. \qquad (A.3)$$

Equation (A.3) may be integrated to give the average temperature over a circular area on the surface

$$T(\text{ave}) = q\theta_s = \frac{q}{2\pi^2\kappa\rho c a^2}\int_0^a\int_0^{2\pi}\frac{r\exp(-V\{\sqrt{[(x+r\cos\theta)^2+(y+r\sin\theta)^2+z_0^2]}-(x+r\cos\theta)\}/2\kappa)\,d\theta\,dr}{\sqrt{[(x+r\cos\theta)^2+(y+r\sin\theta)^2+z_0^2]}}. \qquad (A.4)$$

where a is the radius of the area and x, y, $-z_0$ are the co-ordinates of its centre.

This integral is not analytic, but some approximate numerical values have been obtained for the particular case of $y = 0$ (i.e. for moving point sources passing perpendicularly below the centre of the circle) and these are presented in figure one for various values of the non-dimensional speed ($V' = Va/2\kappa$). The co-ordinates x and z are given as multiples of the radius a and z is measured from the surface to the source. Temperatures are plotted in the form θ_s/θ where θ is the average temperature due to an equal source distributed uniformly over the circle.

The solution for a distributed source could be obtained by a suitable integration of equation (A.4).

CONDUCTION DE LA CHALEUR À PARTIR DE SOLIDES EN GLISSEMENT

Résumé—Les limitations d'échelle importantes du flux de chaleur à partir de deux solides en glissement peuvent avoir un grand effet sur le champ de température près de l'interface. On montre qu'un système pratique peut être représenté d'une façon approchée par deux solides semi-infinis dont les températures à l'infini sont reliées aux flux de chaleur à travers eux. Un grand nombre de solutions existantes pour des solides semi-infinis ont été généralisées pour tenir compte d'une différence entre les températures à l'infini et une nouvelle solution particulière est élaborée dans le cas d'une production de chaleur sous la surface. La méthode est alors étendue aux situations avec plusieurs surfaces de contact et l'effet des propriétés géométriques et physiques sur les conditions aux limites interfaciales est discuté.

DIE WÄRMELEITUNG IN BEWEGTEN KÖRPERN

Zusammenfassung—Die zahlreichen Widerstände für den Wärmestrom zwischen zwei gleitenden Körpern können einen erheblichen Einfluss auf das Temperaturfeld nahe der Zwischenschicht haben. Es wird gezeigt, dass ein praktisches System durch zwei halbunendliche Körper approximiert werden kann, deren Temperaturen im Unendlichen dabei in Beziehung mit der Grösse des sie durchdringenden Wärmestroms stehen. Eine Anzahl bekannter Lösungen für Halbunendliche Körper wurde verallgemeinert um einen Unterschied der Temperaturen im Unendlichen zu berücksichtigen und ausserdem wird eine neue partikuläre Lösung für den Fall der Wärmeerzeugung unterhalb der Oberfläche entwickelt. Die Methode wird erweitert auf Anordnungen mit mehreren Kontaktflächen und der Einfluss von geometrischen und physikalischen Grössen auf die Grenzbedingungen der Zwischenschicht wird untersucht.

ПЕРЕДАЧА ТЕПЛА ТЕПЛОПРОВОДНОСТЬЮ ОТ СКОЛЬЗЯЩИХ ТВЕРДЫХ ТЕЛ

Аннотация—Исследовалась передача тепла теплопроводностью при скольжении твердых тел большого размера. Показано, что практическую систему можно аппроксимировать двумя полубесконечными твердыми телами, температура которых в бесконечности относится к тепловому потоку, проходящему через эти тела. Обобщен ряд имеющихся решений для полубесконечных твердых тел с целью учёта разности температур в бесконечности и разработано новое частное решение для случая генерирования тепла под поверхностью. Затем метод был применен к случаям с несколькими контактными площадями и рассмотрено влияние геометрических и физических свойств на граничные условия на поверхности раздела.